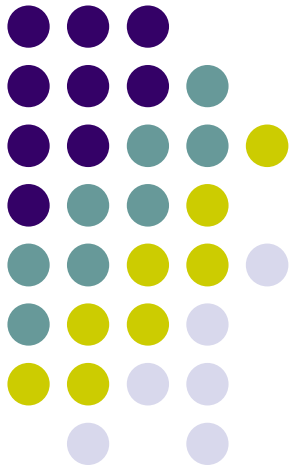
Cavity Resonators

Microwave Engineering

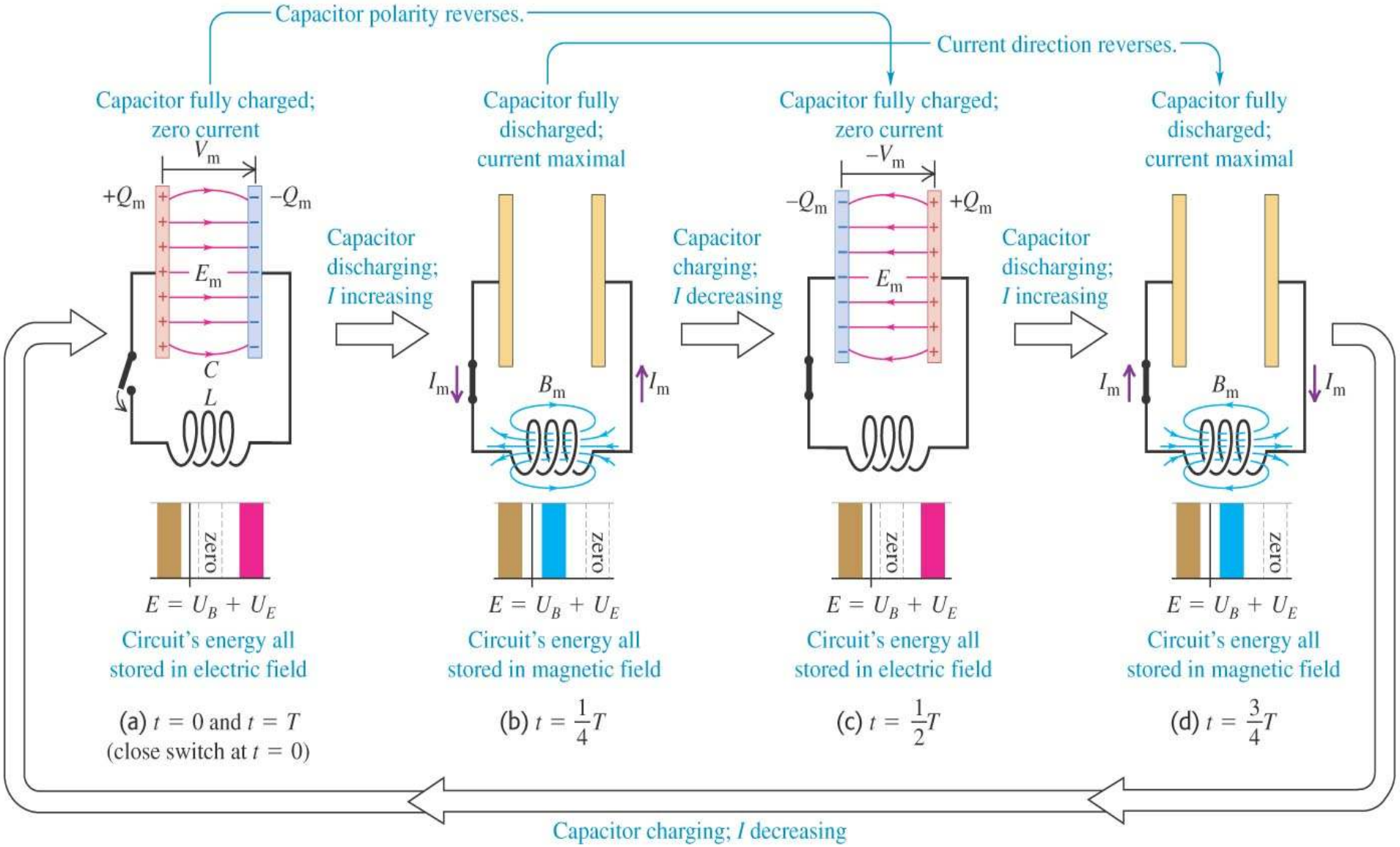
EE 172

Dr. Ray Kwok

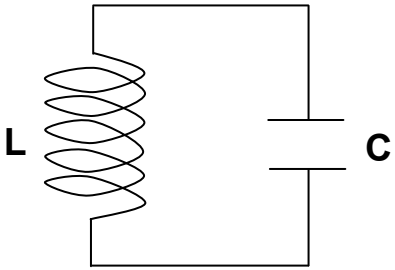


Reference: Feynman, Lectures on Physics, Vol 2

LC Resonator (Lenz's Law)

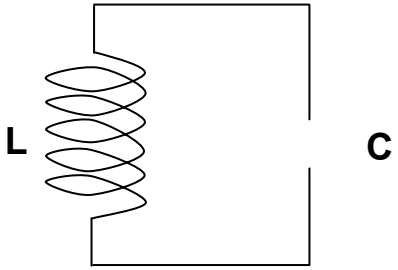


Helical Resonator

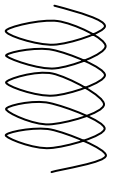


$$\omega_o = \frac{1}{\sqrt{LC}}$$

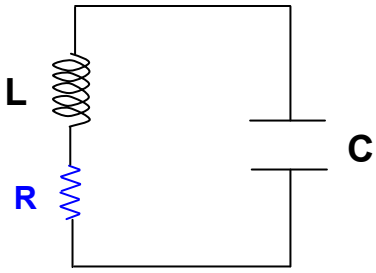
Higher frequency → smaller L or C



smaller C → smaller area

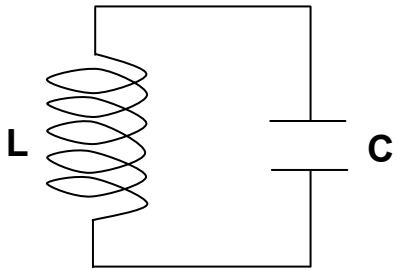


Just the coil itself – resonate
(Helical Resonator)
Internal capacitance between turns
Can't use coil in very high frequency

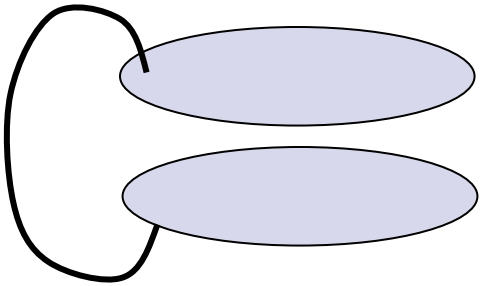


real coil equivalent

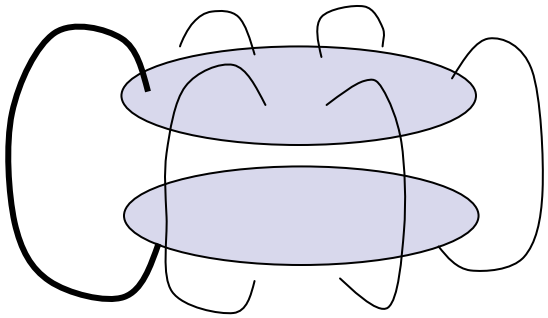
Cavity Resonator



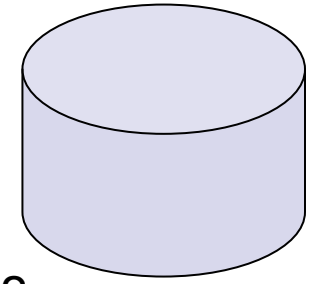
$$\omega_o = \frac{1}{\sqrt{LC}}$$



smaller L → less turns

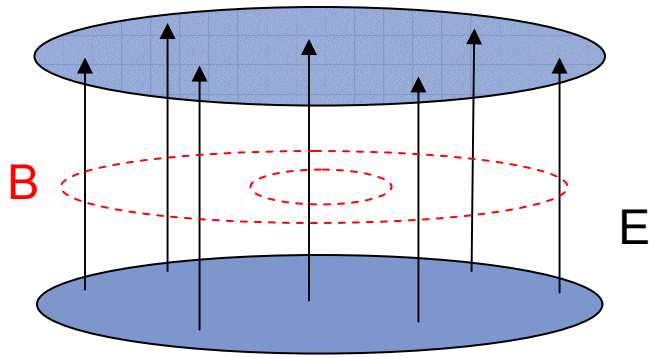


even higher f → parallel L

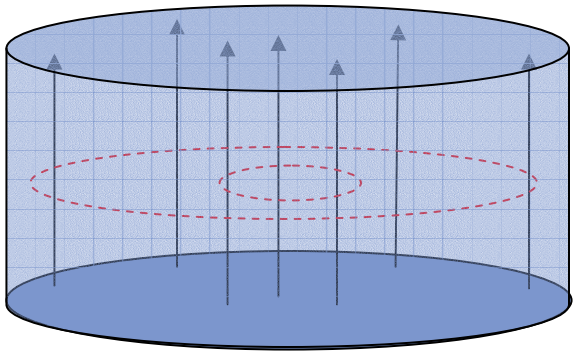


both E & B resonate inside?

High frequency capacitor

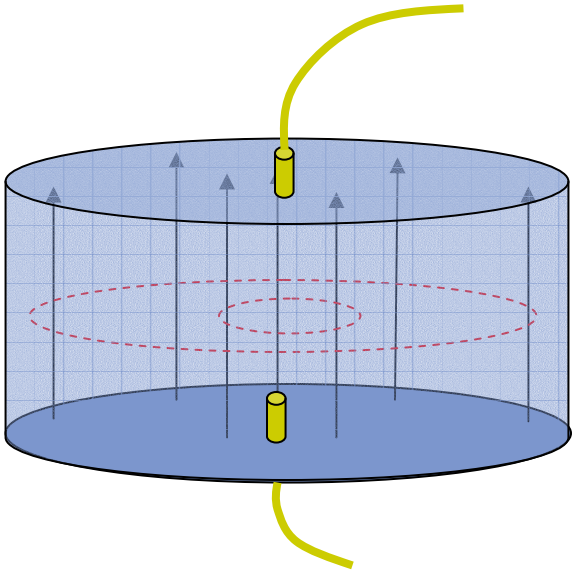


dc – no B
ac – E & B coexist

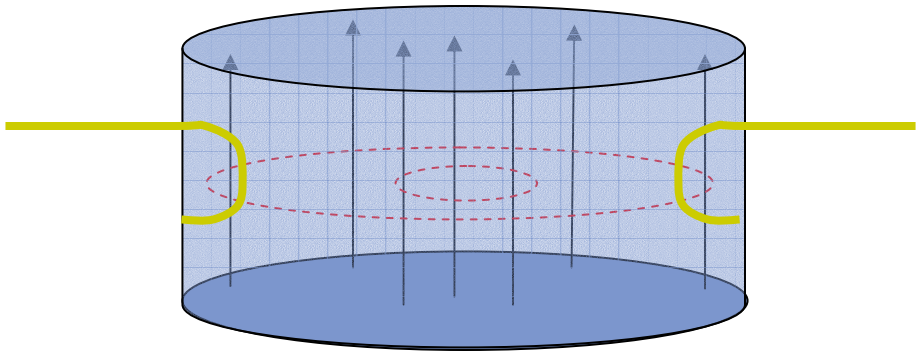


cavity – except tangential E = 0 on the walls,
more field strength at center ….etc

Coupling in and out of cavity

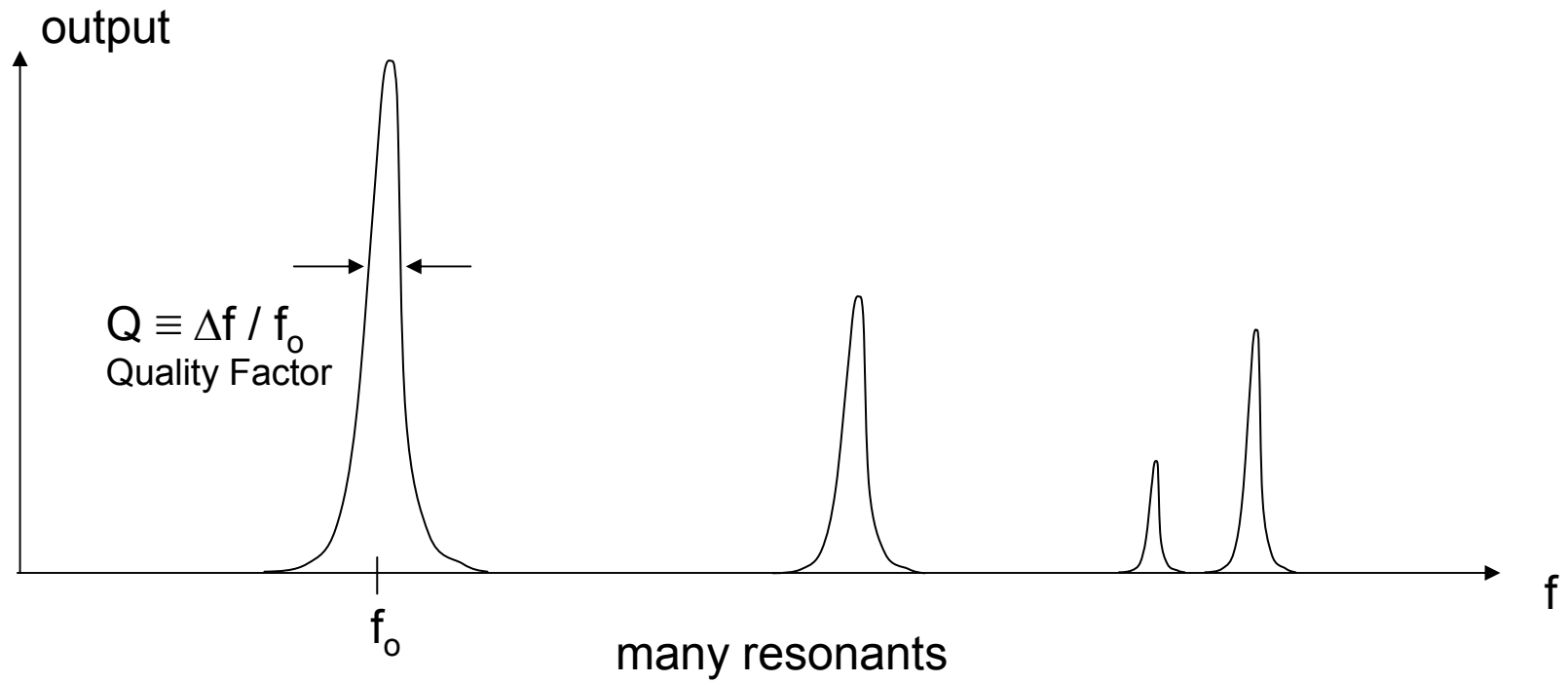


Wire / connector
Couple E-field
Capacitive coupling
Line up with concentrated E-field to induced V

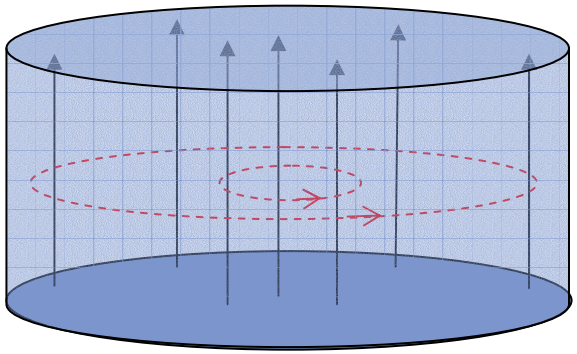


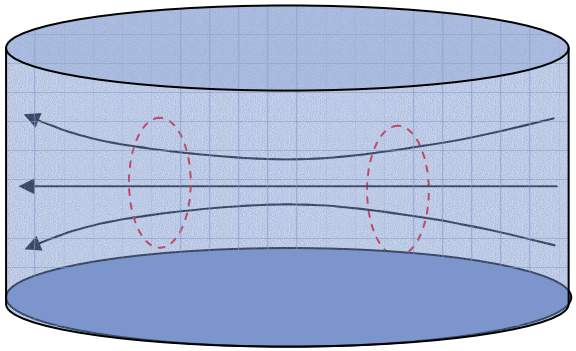
Wire / loop
Couple H-field
Inductive coupling
Loop thru H-field to induced current

Resonate Frequencies

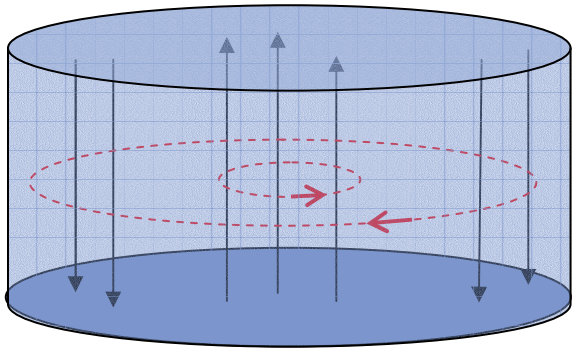


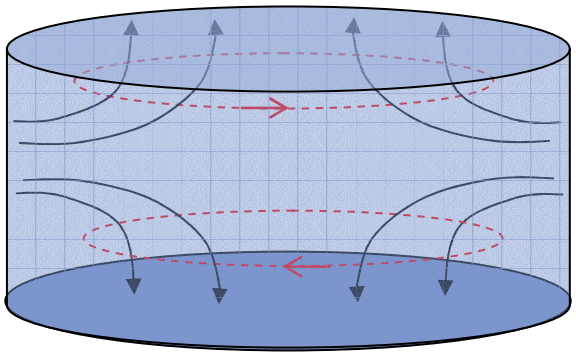
Different modes



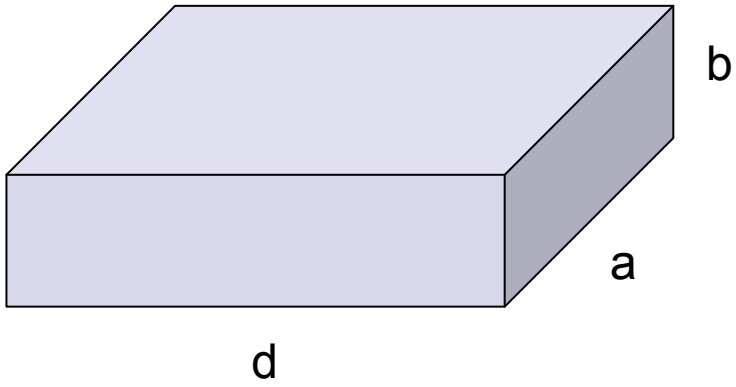


some require different coupling mechanisms





Rectangular Cavity Resonators



$$k^2 - \beta^2 \equiv k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$k^2 = \left(\frac{2\pi}{\lambda}\right)^2 = k_c^2 + \beta^2$$

$$d = \frac{p\lambda_g}{2}$$

$$\beta = \frac{2\pi}{\lambda_g} = \frac{p\pi}{d}$$

$$k_{mnp}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

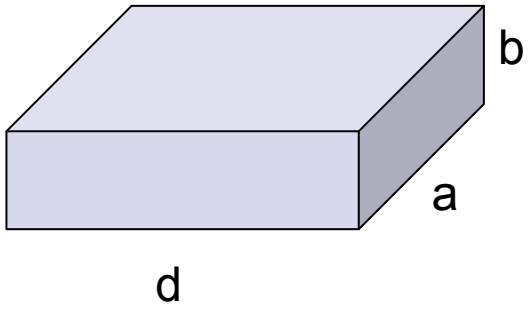
$$2\pi f_{mnp} = c k_{mnp}$$

For TE_{mnp} and TM_{mnp} modes

$$f_{mnp} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

air

TE_{mnp} modes



$$E_z = 0$$

$$E_x \sim \sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right)$$

From boundary conditions

$$E_y \sim \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{p\pi}{d}z\right)$$

$$m, n = 0,1,2....$$

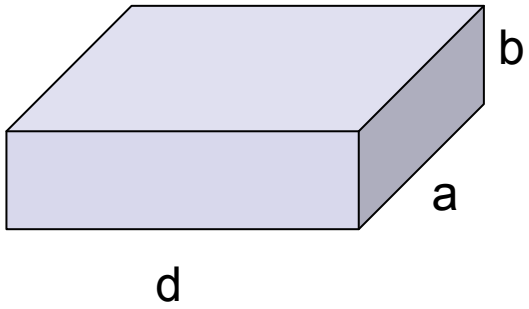
$$p = 1,2,3...$$

m & n cannot be both 0 as in the waveguide, p cannot be 0 !!

First cavity mode is TE_{101}

But a, b, d are interchangeable !!!!
So be careful when labeling the modes!!

TM_{mnp} modes



$$E_z \sim \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$E_x \sim \sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right)$$

$$E_y \sim \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{p\pi}{d}z\right)$$

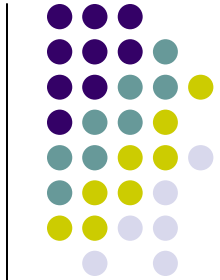
From boundary conditions

$$m, n = 1,2,3....$$

$$p = 0,1,2,3...$$ p can be 0.

First cavity TM mode is TM_{110}

Again a, b, d are interchangeable !!!!



Example

a 6.75 inches

b 5.6 inches

d 9 inches

m	n	p	f (GHz)	mode
1	0	1	1.093611	TE101
0	1	1	1.24203	TE011
1	1	0	1.370224	TM110
1	1	1	1.519233	TE/TM111
1	0	2	1.577228	TE102
0	1	2	1.683539	TE012
2	0	1	1.868763	TE201
1	1	2	1.897296	TE/TM112
2	1	0	2.042989	TM210
0	2	1	2.20882	TE021
1	2	0	2.283367	TM120

a 9 inches

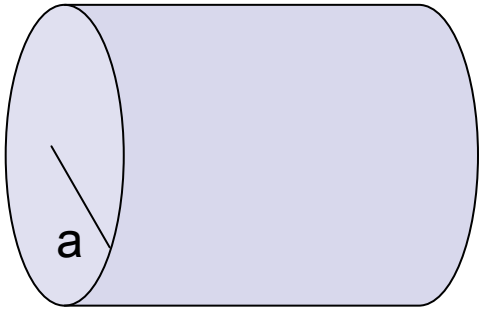
b 6.75 inches

d 5.6 inches

m	n	p	f (GHz)	mode
1	1	0	1.093611	TM110
1	0	1	1.24203	TE101
0	1	1	1.370224	TE011
1	1	1	1.519233	TE/TM111
2	1	0	1.577228	TM210
2	0	1	1.683539	TE201
1	2	0	1.868763	TM120
0	2	1	2.042989	TE021
1	0	2	2.20882	TE102
0	1	2	2.283367	TE012
1	1	2	2.375777	TE/TM112

Same cavity, same set of resonant frequencies. Just different notation.
Not all modes can be excited.
The probe connection dictates which orientation is correct !!

Cylindrical Cavity Resonators



$$k^2 = \left(\frac{2\pi}{\lambda}\right)^2 = k_c^2 + \beta^2$$

$$\beta = \frac{2\pi}{\lambda_g} = \frac{p\pi}{d}$$

$$k_{nmp}^2 = k_c^2 + \left(\frac{p\pi}{d}\right)^2$$

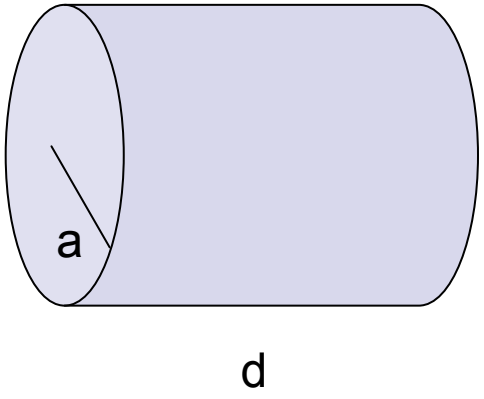
$$f_{nmp} = \frac{c}{2\pi}\sqrt{k_c^2 + \left(\frac{p\pi}{d}\right)^2}$$ air

e.g. Coke can, a ~ 1.25”, d ~ 5”

TE_{111}: $k_c = 1.8412 / 1.25 = 1.473$

$$f_{111} = \frac{11.811}{2\pi}\sqrt{(1.473)^2 + \left(\frac{1\pi}{5}\right)^2} = 3.01\text{GHz}$$

TE_{nmp} modes



$$E_z = 0$$

$$E_\rho \sim (A\cos n\phi - B\sin n\phi) J_n(k_c\rho) \sin\left(\frac{p\pi}{d} z\right)$$

$$E_\phi \sim (A\cos n\phi + B\sin n\phi) J_n'(k_c\rho) \sin\left(\frac{p\pi}{d} z\right)$$

$$J_n'(k_{nm}a) = 0$$

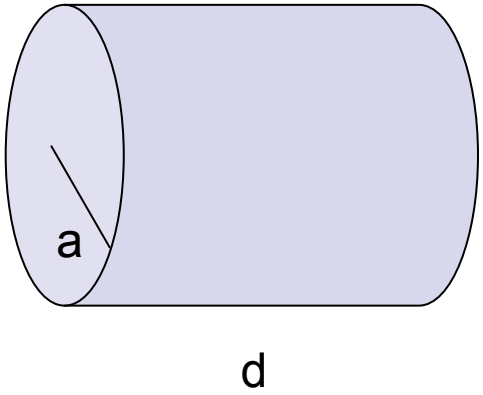
From boundary conditions.

$$p = 1,2,3\ldots$$

p starts from 1

First TE cavity mode is TE_{111}.

TM_{nmp} modes



$$E_z \sim (A\cos n\phi + B\sin n\phi) J_n'(k_c\rho)\cos\left(\frac{p\pi}{d}z\right)$$

$$E_\rho \sim (A\cos n\phi + B\sin n\phi) J_n'(k_c\rho)\sin\left(\frac{p\pi}{d}z\right)$$

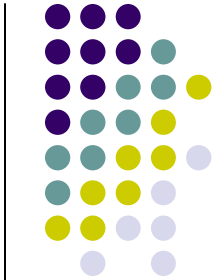
$$E_\phi \sim (A\cos n\phi - B\sin n\phi) J_n(k_c\rho)\sin\left(\frac{p\pi}{d}z\right)$$

$J_n(k_{nm}a) = 0$ From boundary conditions.

$p = 0,1,2,3...$ p begins at 0.

p = 0 means E_r and E_ρ = 0 !!!
And cannot be excited with connector on the sides!

First TM cavity mode “usually” is TM_{011}.



Example

a = 1.9”
d = 6.82”

TE

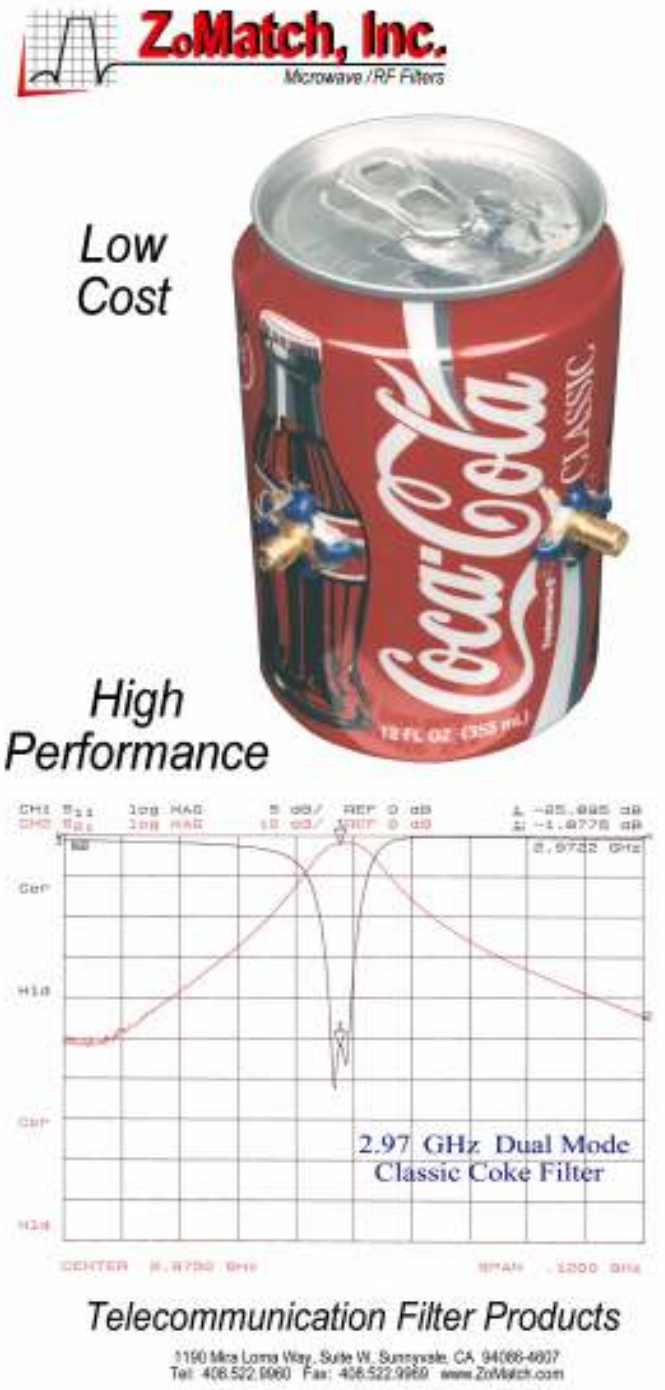
n	m	p	f (GHz)
1	1	1	2.016756
1	1	2	2.513305
2	1	1	3.14312
1	1	3	3.172648
2	1	2	3.482614

TM

n	m	p	f (GHz)
0	1	0	2.379399
0	1	1	2.532062
0	1	2	2.942912
0	1	3	3.522744
1	1	1	3.888838

Again, not all modes can be excited.

Resonant



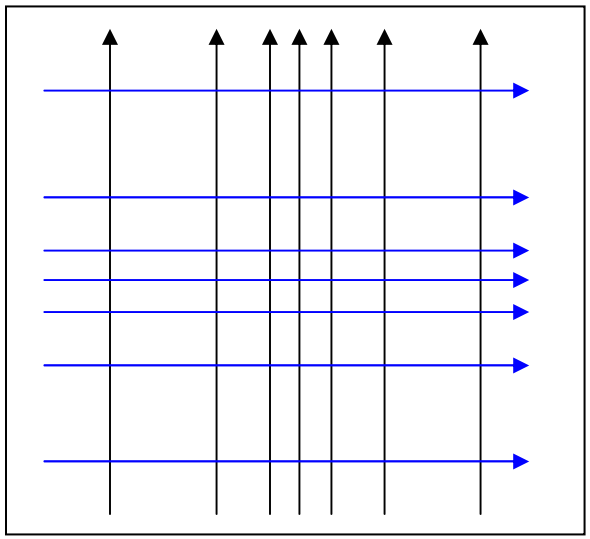
e.g. Coke can, a ~ 1.25", d ~ 5"

TE_{111}: $k_c = 1.8412 / 1.25 = 1.473$

$$f_{111} = \frac{11.811}{2\pi}\sqrt{(1.473)^2 + \left(\frac{1\pi}{5}\right)^2} = 3.01\text{GHz}$$

Dual Mode Cavity

e.g. TE_{10}

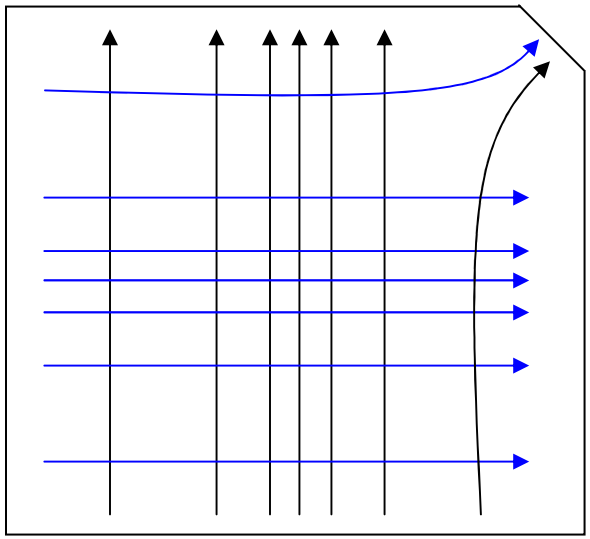


square waveguide

orthogonal

Perturbation

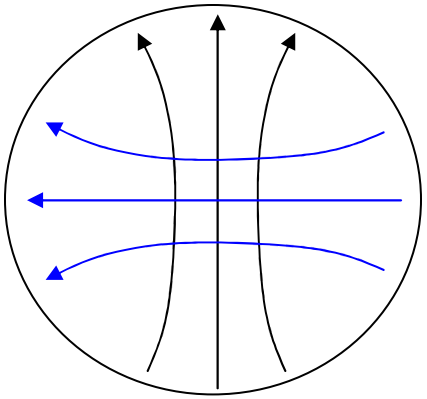
e.g. TE_{10}



coupled modes

Use for:
Circular polarization
Dual cavity
Cross-coupled

Dual Mode



TE_{111} mode

Up to 5-modes cavity has been demonstrated in a spherical cavity.